INTRODUCTION TO «THE SCIENTIFIC DEMONSTRATION»*

BRUNO LEONI

In 1955 Bruno Leoni, the President of the Methodological Study Centre of Turin, together with the mathematician Eugenio Frola wrote an essay with the title *Possibility of the application of mathematics to economic disciplines*. It is the start up of an intellectual exchange destined to continue over time, as three years later, on the 20 December 1958, the then President presents to the Centre members the meaning of the work which kept him busy throughout the course of that year with Frola. The Centre was constituted by the meeting of colleagues and friends for a reflection and an exchange of ideas and positions regarding the issues of method, considered under various aspects and regarding a plurality of intellectual disciplines: from the mathematic analysis, to logic; from the history of philosophy, to physics; from the new Geymonat's rationalism of philosophy of science, to the Austrian School, and so forth. Among the Centre members, important names of the 20th Century culture appear: from Ludovico Geymonat to Nicola Abbagnano; from Enrico Persico to Piero Buzano, from Prospero Nuvoli to Norberto Bobbio.

The here published work reconstructs the background of the problems which, during his work with Frola, Leoni has taken to heart, his doubts in the dialogue with his colleague, the questions he posed, so showing the live progress of his thought. The «by tentative» procedure immediately emerges with which the two scholars applied themselves the work. There is a sign of concreteness linked to Leoni's empiricism. Instead of attempting a generic «philosophy of demonstration», the two scholars examine particular examples, which recall demonstrations historically occurred. They cite Legrange, Cantor, Hilbert, Ricardo and so forth. They observe the difference with which there is sometimes between the value that the author attributed to his results and that one recognised by the scientific community.

Throughout the research, two different tendencies were outlined. Frola starts from an interpretation totally «convenionalist» of the demonstrative procedure. In his opinion, it is conventional: the starting

* Leoni's speech. 20 December 1958 at the Methodological Study Centre of Turin.

Procesos de Mercado: Revista Europea de Economía Política
Vol. VI, n.º 2, Otoño 2009, pp. 291 a 301

point, the singular logic used to derive a conclusion from the premises, and the final demonstrations. One could say that, for Frola, the demonstration consists in the definition of an analytic «truth», a tautology. On the other hand, for Leoni, the method followed in the demonstrations starts undoubtedly from conventional premises and develops according to certain rules which in turn could be considered conventional, but, after undertaking these premises and setting the rules of demonstration, the conclusion reached – the «*ergo*» which completes the demonstrative «*iter*» —is not conventional, as when we string together premises among each other— «bound» together, according to Frola's expression in the course of process.

Leoni insists on the cogence with which, in a demonstration, the conclusion follows its premises. A certain way of concluding is evoked which recall the deductive method, making a rule of the syllogism's logic. And it is clear that in this perspective, a different way of concluding, of «binding» the premises cannot subsist: since, if by means of observation, I discover that A is more than B, and that B is more than C; A cannot be other than more than C. The conclusion is inexorably the same, whoever the scholar studying this last passage, this «bond».

It is useful to notice, nevertheless, how the two opposing views are kept inside the area in which there is an consent. Leoni does not entirely repudiate the relativism of the demonstrations, but only the idea that every phase of a demonstration is «conventional». Therefore, not a general criticism of conventionalism, but rather that conventionalism must always be considered as an arbitrary process, without significant relations with the reality of the outside world.

It emerges in this context, the importance attributed by Leoni to the lexicographic analysis, controversially proposed against the formalist nominalism; and here the author accomplishes that «quality jump» which one has when «the general theorist transforms himself, finally, in critic of his own language»[1]. Leoni invests himself in the historical-lexical research, to «see if there exists a minimum common significance»[2] of the term demonstration; and here he finds reasons for his anti-conventionalist theory. In fact, he observes how the term has the prevalent meaning to show —*(de)monstrare*— something which exists independently, therefore assuming a meaning similar as to the «to discover», and therefore sketching a theory of evidence, which nearly seems to match the empiricism and the catallaxy, concept linked to the Hayekian irrational individualism.

---

[1] Uberto Scarpelli, «Bruno Leoni and the analysis of language», *Il Politico*, XLVII, 1982, n.º 1, p. 138.

[2] *Ibidem.*, p. 140.

With this regard, it is interesting the reference to the game of chess and to the studies of G. Abrahams. Here Leoni, recalling the division between «analytical» chess players and «intuitive» players, observes how the analytical-empirical study of all the possible cases and of the relative countermoves achieved its rewards, at this time, in the «openings» and in the «closings»: the phases of the game in which the variables to study are less and more easily calculable by the human mind. Thus remains, for the «intuitives», to reveal their talent during the intermediate phase of the match. It is indeed too evident here the reference to the economic science and to the perspective turnaround which was impressed by Carl Menger; but one could also grasp an indirect way of intervention in the debate between deductivists and inductivists, both involved in the endless struggle for the supremacy in the scientific methodology. On this point, one can gather all the credit which Leoni acknowledges to the inventive talent, to the intuition which overturns the antecedent knowledge; but the conviction remains firm that the subject of intuition could really become «knowledge» or «science» only when a demonstration is given according to a rigorous and controlled procedure.

All in all, as the reader will see, in the informal style of a conversation among colleagues and friends, imbued with references which give a sense of common undertaking and, in this field, of an intense intellectual exchange which personally involves Leoni and Frola. But this short writing also brings the confirmation of a style of thought, that which the elements shape the major works of Bruno Leoni: the passion for knowledge, a conscious empiricism, the rigour of the analytical method.

Adriano Gianturco Gulisano

THE SCIENTIFIC DEMONSTRATION

BRUNO LEONI

We have started our work, Frola and I, in a let's say, «tentative» manner. First of all, our concern was to examine some «demonstrations» already effected by both mathematicians (and this —as you know— was just Frola's field) and by scholars of other fields, for e.g. of economics (with particular regard to some classic argumentations, such as Ricardo's theorem of compared costs), of law and so on, which we could consider in comparison with mathematics demonstrations. In other words, rather than attempting a generic «philosophy of demonstration», Frola and I proposed ourselves to concretely examine certain examples of «demonstrations» historically happened, similarly examining —for e.g.— the chess matches played, and seeing what happened, or rather what *was done* in these demonstrations: from which premises the author initiated; what is *mainly*, the area of means that he considered known; what again he has introduced and how he has conducted the demonstration; namely what «*iter*» his reasoning has covered.

We have done this, first of all, by taking as example Lagrange's «Théorie des functions analytiques»[3] (creator of the theory of analytical functions, who moreover has given us an obscure definition of «function»: «It is defined function of one or more quantity, every expression of calculus in which quantities enter in any manner, mixed or not with other quantities considered as having finished values and invariables. While to the function's quantities can be assigned all possible»).

In this reading I was obviously a disciple of Frola. But dare I say that the fact I was inexpert of mathematics was somewhat useful as it led me to challenge my mathematician friend with observations, clarification requests, objections, «why», which effectively stimulated our analysis of the mathematics «demonstration». We then moved

[3] Joseph-Louis Lagrange (1736-1813). Was one of the greatest mathematicians of his century. His *Theory of Analytical Functions* (1797) gathers reflections matured over the past 25 years and, although it was written as a textbook for his students at the École Polytechnique, it soon became a classic. The main idea was to give a solid fundament to the mathematical analysis, releasing it from any intuitive reference to geometric or physics evidence - Ed.

to examine Cantor's «Set Theory» and hence examined Hilbert's treatise[4].

While examining what these authors did, we made a series of observations: we considered for e.g. what were the premises from which they started, we noticed that certain authors —like in the case of Lagrange— considered as a general a solution which was instead a particular one, proposing definitions deliberately general, thereafter the development of science in that sector has considered valid only for some cases of the general theory. Particular cases were generalized and it was believed to enunciate for e.g. the general theory of functions as Lagrange did, while it was enunciated the theory of *particular cases* of functions. The opposite happened to the economist Ricardo: he has enunciated a theorem (that of comparative costs)[5] the validity to which was considered by subsequent economists more general then he retained, considering his theorem valid only for a particular case, that of international exchanges.

Evidently, during the examination of these demonstrations we came up against an apparent lack of «rigour». Lack of rigour above all in the use of words, in the bargain obtained from common language, whereby the general definitions were given and from which we then started the demonstration (we notice this especially in Cantor and in Lagrange, a bit less in Hilbert). It was seen how these authors, when giving definitions, which are the starting points of all their reasoning, state fundamentally things not completely comprehensible, inasmuch as they use terms without precise meaning and so are not defined, whereupon leaving us perplexed. As Geymonat aforesaid, there evidently exists a historic situation (specifically: in language history) to which these authors refer, or alternatively the particular *forma mentis* of this people emerges, whom sometime pose as philosophers, and even though they have a very

[4] Georg Cantor (1845-1918). The father of «Set Theory», who through his proposed formulation, gave rise to the famous antinomies. Cantor contributed to the introduction of the «actual infinity» notion in modern mathematics as well as the theory of «hierarchy» of infinities, against any intuitive evidence. The «Grundlangen der Geometrie» (Fundations of Geometry), written in 1899 by David Hilbert (1862-1943) proposes an axiomatic arrangement of the Euclidean Geometry in the spirit of modern conception of the axiomatic systems which gradually evolved in the second half of the century XIX - Ed.

[5] The theorem of Comparative Costs by Ricardo (1772-1823) is presented in the *Principles of Political Economy and Taxation* (1817-1821), and is especially relevant in the area of international trade; which demonstrates that «the imports can be profitable even when *imported goods* can be produced internally at lower costs than abroad» (Joseph A. Schumpeter, *History of Economic Analysis*, Turin, Boringhieri, 1972, p. 272) - Ed.

relative notion of the philosophical language, carelessly borrow certain terms from philosophy to build their definition. This way the philosophical language or pseudo-philosophical acts as a meta-language of the mathematics language (It is shown for e.g. the definition of «set» and the definition of «power» or «cardinal number» in Cantor: «We call «set» every *M* union of objects in our thought *m*, certain and well distinct, and what we would define as «elements» of *M*» - «We call «power» or «cardinal number» of *M*, the general notion which we deduce from *M* with the help of our thinking faculty, abstracting the nature of different elements *m* and their order»).

At times we notice an imprecise language at an amateurish level used in the practice of mathematicians or physicians of a certain period (for e.g. Lagrange's above mentioned definition of function). Certain authors whom nonetheless have an evident intellectual power, and whom establish new theories of great importance, sometimes reveal to our contemporary eyes a remarkable ingenuousness in determining their starting points.

For instance Cantor's set theory requires, as a logic and obvious premise, the notion of *unity,* and in particular the notion of a «distinct» object, therefore, implicitly that of plurality, and finally all notions that are presumption of the natural numbers theory (even if the characteristic of Cantor's theory is the passage to the notion of transfinite number). We arrived at this conclusion by some questions that I posed to Frola: to put in a nutshell, I remembered Peano's attempt to create a «minimum vocabulary», using this expression by Russel's expression. Even Cantor searches for his minimum vocabulary, thus trying to reduce his reasoning to certain concepts which he defines. But what is useful in his reasoning, and that is not defined, is precisely this notion of «distinct object» which is at the basis for the «set» notion. Notion that is of *one*, of *single element*. Cantor thought it was self explanatory (in mathematics demonstration we always tend to eliminate tacit presuppositions and to reduce explicit presuppositions. In spite of this, the attempt does not always succeed, in fact I would say it could never totally succeeds).

Furthermore, Frola and I noticed what one would call the «tricks» of these authors. For e.g. Cantor's «trick» in his set theory consists of reproducing a series of argumentations in a new order of ideas, naturally modified in relation to the new theory, which are already in the cardinal number theory. After having formulated basic concepts, in order to develop them, the inventor of the new theory uses an already existing model, consisting in already known demonstrations in the field of finite numbers.

At a certain point during our study, two orientations were outlined: that of Frola and mine. Frola started from an interpretation which could be called «conventionalistic» from all demonstrative reasoning. I understood that, for Frola, «conventional» is not only the starting point of demonstrative procedure, but also the particular logic used to deduce a conclusion from the conventional premises. All this brought Frola to consider this procedure, this demonstrative method, as wholly based on convention, or at least as such to highlight a strong conventional character which the all procedure could be considered as «conventional». I was not of this opinion, because for me the method we used in analysing the demonstrations, initiated undoubtedly from a «conventional» premise, and furthermore developed according to certain rules that could be also considered as conventional. However, when the author adopted these premises and set the rules for the demonstration, he *had to* and thought that one must arrive at a certain conclusion. Frola disagreed that at a certain point we could alter the premises and moreover alter the demonstration rules; I replied: «Ok, but when you have altered the premises and elaborated "other" rules, at a certain point you must then conclude in a certain "other" way and this will no longer be "conventional". Just the "ergo" at the end of the demonstrative *course*, is not conventional: you yourself have stated your premises, your own rules, but (and I wouldn't say this is a subject in favour of "convention"), given such premises and such rules, you *must* conclude in a certain way. Now, the fact you have to "conclude" in a certain way, given certain rules, is not subject of "convention"». (Aristotle would have called this αναγκαστική δύναμιζ)[6].

This matter was (and is) very important to me as it showed us that the «rigour» is not something conventional; there are *exigencies* which we answer to, and that we try to satisfy in the demonstration, which are not at all «conventional». So what was the «rigour»? This is the point on which I request Frola's attention. He told me: «You have some premises, and you continue to repeat these premises throughout the development of the reasoning (the famous theory to which mathematical reasoning would be tautological) and so if you repeat these premises and «bind» them together, you have the demonstration». But, evidently —I counter replied— this premise repetition takes place «in that certain way». And what is the «binding»? For me the «binding» is something which needs to be analysed, just because the method in putting the premises together is probably not «conventional». In the end, Frola seemed a bit uncertain regarding this point, but the issue remained *sub iudice*.

At the same time, I took pleasure in doing some research on the lexical history of the «demonstration». Geymonat would say that such researches are useless, as today we could have a completely different idea of the

---

6 Which is «coercive strength» or «coercive capacity».

«demonstration», compared to that of the old fashioned one. I, on the other hand, believe that the lexical history researches could be somewhat useful, if for no other reason than to clearly let see how in past times the demonstration was conceived in a certain way which is definitely not conventionalist. I would say that the demonstration etymology «demonstratio», already in Latin, means the «operation to show». Such an operation can only be conceived outside of the conventionalist field: indeed we can rather «conventionalise» the term, with which we mark what we see and with which we inform someone of the presence of an object, but we cannot conventionalise the thing we show. In the «demonstratio» concept, is implicit the idea that in the same way one shows physical objects, you can also «show» conclusions. In other words, the conclusion, and with this I would say Frola's famous «bond», appears in the linguistic analysis of the term «demonstratio» just as an approximate empirical intuition, an «evidence». Even the Latins seemed to agree with my thesis, that the demonstration is not entirely conventional, as it consists in «showing» something which «is».

Looking back, not only at Frola's understanding, but also that of our colleague Mario Vallauri, to see what happened at the origins of the Indo-European languages, we found something similar in the Sanskrit, (interestingly) insofar as in the Sanskrit there are no terms which effectively express this idea of the «demonstration» by a reasoning. But there are terms, such *dìsat*, which correspond to the Greek δεικνύμι (that is to show) —and you know «to show» in Greek is εττιδεικνύμι or αττοδεικνύμι. Therefore the verbs used in Sanskrit, as well as in Greek correspond, for their meaning, to the Latin «demonstrare».

If we examine Germanic languages, we find the same thing: for e.g. the German verbs *anzeigen* or *beweisen*, or the English *show*, all mean «to show» as well as «to demonstrate». Going back to Latin, in Plinio's treatise «De cane venatico» he says: «visa fera quam silens et occulta quam significans *demonstratio* est». Moreover, it is interesting the term «genus demonstrativum» that was a particular type of rhetorical speech which *demonstrated* and thus pointed out the virtues and faults of the people. As was said «digitus demonstrativus». For the Latins, «demonstrare» always had not only the meaning to argue the validation of a conclusion, but also to «show» the «things».

It is interesting the meaning the Roman Jurisconsults give us of the term «demonstratio», a slightly different meaning compared to that of properly demonstrating and arguing. Forcellini notices: «a juris consultis dicitur demonstratio cum res aliqua ita certis quibusdam ac peculiaribus signis designatur ac veluti depingitur ut nullus ambigendi locus reliquatur ut si quis in testamento dicat *decem, quae mihi Titius debet, lego*. Constat enim ea sola decem, quae debet Titius, non aliam pecuniam legari». This is an example which is given to us by Gaius: if I say «I bequeath these ten denarii which Titius and no one else owes me» there is evidently no doubt on the denarii I am bequeathing. The Jurisconsults call this «demonstratio»: there is no doubt whatsoever, because even if I did not point with my finger at the denarii, nor did I show them by reasoning, I nonetheless pinpointed them so precisely as not to leave any doubt. Thus, there are all the equivalent terms: for e.g. «demonstrative» which corresponds to the Greek έττδεικτικώζ, «demonstrativus» which equally corresponds to the Greek έττδεικτικόζ, qui ad demonstrandum aptum est, ut digitus demonstrativus.

Is also interesting another judicial meaning of the word «demonstrare» namely the «demonstrare fines» which is the final relation established between the buyer and the seller of a land: the proceeding in which the land seller showed the buyer the boundaries, he showed them and relinquishes ownership. Therefore, even in judicial language, the «demonstratio» meant to show something. It maintained the same meaning in the Middle Ages: and also the term «demonstramen» («exibitio»), or that «demonstratio cartarum» (displaying documents, especially with proven records of ownership stocks and the like); there is also a curious meaning of the word «demonstratio»: this term pointed out certain kinds of taxation, for e.g. the besenagium. Another topic discussed in our research regards the *chronological priority*, or maybe also the *logic* of the demonstration, on the theorem statement. Frola even considered it was possible that in the reversal were mythical reasons. A very interesting point is: the relation between theorem and demonstration. Usually (something which always amazed me when I was a high school student) we were told a theorem to which the origination is often unknown, then comes the demonstration. It appears to be a kind of magic trick. This is however not the way the theorem was historically born. The demonstration probably exists before the theorem.

Frola and I also tried to compare that which can be called «discovery» in mathematics and that which can be called «discovery» in geography. We tried to make comparisons between the game of chess and a mathematical demonstration. As for mathematics, there are two schools of thought for chess players; the analytics and the intuitive. Certain players have studied analytically with the demonstrative method, the possible cases which are obviously endless, but there are extremely patient people and with excellent memories who have examined for every move, the possible countermove and for every countermove, the possible countermoves, and doing so for a considerable number of cases. In this way the opening analytical theory in the chess games is by now so consolidated that, given certain initial moves, the countermoves can be considered obligatory. Today we know, for e.g., that certain openings which we have been considering valid for fifty years, then revealed such defects, that these openings are now advised not to adopt.

The study of chess openings is therefore now consolidated: it is analysis, it is theory: the theory has furthermore taken possession of the conclusions and it is now known that when the pieces are reduced to just a few, the game has to take place in a certain way because otherwise one would lose, and therefore he who knows the conclusions theory is superior to the opponent who doesn't know it. What is still an open field is the middle of the game, because here the game depends on such a large amount of combinations that it has not been able to be invaded by analysis, if not only partially. And here who obviously play the synthetics, the inventives, or if you like, the intuitives, who think that the right move is the one they are playing, but they cannot demonstrate this as the analysis has not yet been made. I believe we have a conceptual situation here very similar to that of certain mathematical fields in which a demonstration is sought.

Some players have a particular genius in imagining, without giving any demonstration, what moves can lead them to success, and only by being patient, and frequently for a long period, the analytics, can bring to demonstrate that the move was correct, or alternatively ingenious but defective. It seems to me that the mathematical process is reflected here, when looking for a demonstration with the method of the inventive player, thus leaving to the analytics to «manage» this demonstration, by accepting it, or by refusing it if it results incorrect. Some interesting books were recently published on this subject, for e.g. by G. Abrahams, an English author who studied the mentality of the jurist and that of the chess player in two distinct works («The legal mind» and «The Chess mind»)[7].

As I was saying, we then studied the meaning of «discover» in mathematics, because effectively «discover» implicates the idea of something that «was» and that no one has yet seen. (For me this is another point in favour of the evidence theory, in which one can therefore not reduce all to convention). In what sense do we say in mathematics that one discovers something? Do we see something that «was already there», or have we «created» or «invented» something? Does there exist a conceptual relation between this «discovery» and the «discovery» of a geographical territory?

We additionally noticed, Frola and I, how in the demonstration, different possible logics intervene, some which are privileged in the sense

[7] Gerhard Abrahams, *The Legal Mind: An approach to the Dynamics of Advocacy*, London, H.F.L., 1954; Id., *The Chess Mind*, London, English Universities Press, 1951 - Ed.

that they are commonly used (today for e.g. the acceptance of Zermelo's[8] postulate is rare), as do exist geometries commonly used (for e.g. in classical physics).

We submit to you, dear and illustrious colleagues, a series of hints for our subsequent work and maybe our work will continue, in fact I certainly hope so, as soon as Frola's health is re-established[9].

---

[8] The Zermelo's postulate (Ernst Zermelo, 1871-1953) often marked as the «axiom of choice», has various equivalent formulations. One of these is the following: for any set A whose elements are also non-empty sets, exists a set B which has only one element in common with each element of A. This affirmation is obviously interesting when they are infinite sets. Other formulations contemplate the Cartesian products of sets or the concept of functions (choice function). This axiom aroused bitter controversy among the mathematicians creating different trends according to the acceptance or not of such postulates. Today, this problem is overcome and the axiom is generally accepted (implicitly) in the mathematicians' practice - Ed.

[9] Addition by Prof. Eurgenio Frola: «I am perfectly in agreement with Leoni on certain things, but on others, I do not agree as evidently it is very difficult to understand each other in these things and hence I do not entirely approve the translation of my language, in the Leonian language. For example: I do not agree with Leoni's idea on the demonstrative value which would have for its anti-conventionalist thesis, the meanings of the word "demonstration" in the Indo-European languages. Besides, the investigation is incomplete, we did not refer to the same period and we did not go back to the one which eventually could be of interest, the epoch when the Indo-European people could be united, and therefore, could have not only one language, but a common civilisation».