MISES, MARSCHAK, AND MARKOWITZ: THE SOCIALIST ORIGINS OF MODERN PORTFOLIO THEORY

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Fecha de recepción: 30 de diciembre de 2022
Fecha de aceptación: 1 de abril de 2023

Resumen: Este artículo estudia que la Teoría Moderna de Cartera surgió de la tradición socialista a raíz del debate sobre el cálculo socialista. Tras el debate académico con Mises, el socialista Marschak desarrolló los elementos básicos de la Teoría Moderna de Cartera y se los transmitió a Markowitz. Marschak y Markowitz desarrollaron la Teoría Moderna de Cartera basándose en la teoría subjetiva de la probabilidad. Su enfoque subjetivo debe ser rechazado, ya que se basa en la filosofía autorrefutante del relativismo individual. Además, la Teoría Moderna de Cartera no puede reconciliarse con la correcta teoría frecuencial de la probabilidad. La Teoría Moderna de Cartera forma parte de la revuelta socialista contra la razón y, por tanto, está destinada al fracaso científico.

Palabras clave: Teoría Moderna de Cartera, Socialismo, Teoría Subjetiva de la Probabilidad, Teoría Frecuencial de la Probabilidad, Jacob Marschak, Harry Markowitz

Clasificación JEL: B24, G00, G11

Abstract: This paper shows that Modern Portfolio Theory emerged from the socialist tradition in the aftermath of the socialist calculation debate. After opposing Mises in the debate, the socialist Marschak developed the basic elements of Modern Portfolio Theory and handed them down to Markowitz. Marschak and Markowitz developed Modern Portfolio Theory on the basis of the subjective theory of probability. Their subjective approach must be rejected, for it is based on the self-refuting philosophy of individual relativism. Moreover, Modern Portfolio Theory cannot be reconciled with the correct frequency

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theory of probability. Modern Portfolio Theory was part of the socialist revolt against reason and was therefore destined to be a scientific failure.

Keywords: Modern Portfolio Theory, Socialism, Subjective Theory of Probability, Frequency Theory of Probability, Jacob Marschak, Harry Markowitz

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> "The historian must never forget that the most momentous occurrence in the history of the last hundred years, the attack launched against the universally valid science of human action and its hitherto best developed branch, economics, was motivated from the very beginning not by scientific ideas, but by political considerations." (Ludwig von Mises 1933, 3)

1. **Introduction**

The year 2022 marks the hundredth anniversary of Ludwig von Mises's masterpiece *Socialism: An Economic and Sociological Analysis* (Mises 1922). This was the classic work of the socialist calculation debate, and it is one of the most "profound" economics books of the twentieth century (Hayek 1978, xix). Also, the year 2022 marks the seventieth anniversary of Harry M. Markowitz's famous paper "Portfolio Selection" (Markowitz 1952). This paper ignited the Modern Portfolio Theory revolution in finance. Today the tools of Modern Portfolio Theory are used to manage "tens of trillions of dollars' worth of assets" (Markowitz 2016, 257).

Mises's *Socialism* and Markowitz's "Portfolio Selection" are not normally associated. However, Modern Portfolio Theory can only be fully understood within the context of the socialist calculation debate. Jacob Marschak is the essential link between Mises and Markowitz. Marschak was one of Mises's opponents in the socialist calculation debate (Marschak 1924). Later he served as Markowitz's dissertation advisor at the University of Chicago. His work had an enormous influence on the young Markowitz. Although Markowitz (1999, 5) is the self-described "founder of Modern

Portfolio Theory," Marschak has a strong claim to this title.[1] Perhaps it is most appropriate to say Marschak and Markowitz are the co-founders of Modern Portfolio Theory (Arrow 1983, 10; Read 2012a, 18).

This paper illustrates that Modern Portfolio Theory was born out of the socialist tradition in the fallout of the calculation debate. The next section shows Marschak and Markowitz were associated with the market-socialist movement. Section three explains that Markowitz adopted two fundamental concepts of Modern Portfolio Theory from Marschak: (1) mean-variance analysis and (2) the subjective theory of probability. The fourth section demonstrates that the subjective theory of probability is a form of relativism and is thereby self-refuting. Section five shows that Modern Portfolio Theory is incompatible with the correct frequency theory of probability. The concluding section argues Modern Portfolio Theory is part of the socialist revolt against reason.

2. **Marschak, Markowitz, and Socialism**

Jacob Marschak was born in Kiev in 1898. He read *The Communist Manifesto* in the summer of 1915 and recalled, "The readings and discussions of the summer made me, then, a Marxist…. I now became a propagandist myself" (1971, 15). He was a "Menshevik revolutionary" during the Russian Revolution of 1917 and then he "led the Mensheviks and Socialist Revolutionaries" of the Terek Soviet Republic (Arrow 1979, 501; Dimand and Hagemann 2019, 3). Like many Menshevik socialists, he fled during Vladimir Lenin's Red Terror. He arrived in Weimar Germany in January 1919 and received a Ph.D. there in 1922.

Marschak's first academic paper was published in 1924 under the title "Economic Calculation and the Commonwealth" (Marschak 1924). As its title makes clear, the paper was a response to

[1] Also, Markowitz is advertised as the "Founder of Modern Portfolio Theory" on the covers of his multi-volume work *Risk-Return Analysis: The Theory and Practice of Rational Investing* (2013; 2016; 2020).

Mises's seminal article "Economic Calculation in the Socialist Commonwealth" (Mises 1920). Harold Hagemann explains,

> "Marschak addresses the problem of valuation of goods of higher order which Mises had considered as impossible to solve in socialism since market prices for the means of production don't exist…. [Marschak's] key argument is that even in the case of complete vertical integration (concentration of all intermediate stages of production), in which an input of labour would lead to an output of consumption goods, the imputation problem can be solved and no difficulties in the determination of prices and quantities would arise…. Marschak criticizes Mises for mixing up the question of valuation of goods of higher order with the impossibility of economic calculation in a socialist commonwealth characterized by economic change, i.e. dynamics, and thereby mixing a purely theoretical with a practical question. Furthermore, Marschak argues that both questions, the economic calculation for goods of higher order as well as the dynamic problems, show the inadequacy of Mises's model of capitalism." (2019, 229–30)

Mises was certainly familiar with Marschak's paper. He wrote, "Jacob Marschak attempts to get at the problem by criticizing economic calculation under the social system that is based on private ownership. He says that economic calculation under capitalism does not provide an accurate calculus of value" (1928, 367). Mises's point here is significant: Marschak was hostile to classical finance. As will be seen, Markowitz adopted Marschak's anti-finance mentality and thus attempted a radical "reformulation" of financial science (Markowitz 1952, 91).

After 1924 Marschak collaborated with Emil Lederer, "the leading academic socialist of Germany in the 1920s" (Schumpeter 1994, 884n10). Together they produced a standard socialist text entitled *The New Middle Class* (Lederer and Marschak 1926; Lederer and Marschak 1937). Marschak moved to the University of Oxford in 1933 with the help of an eccentric communist named Redvers Opie (Arrow 1979, 502).[2] At Oxford, Marschak hosted the famous

[2] There are reasons to suspect Opie was a Soviet spy (Young and Lee 1993, 204). In 1939, Keynes installed Opie as the British Treasury's representative in Washington

conference on September 26, 1936, where Roy Harrod (1937), James Meade (1937), and John Hicks (1937) presented their famous Keynesian IS-LM papers. Warren Young rightly describes Marschak's IS-LM conference as "probably the most significant conference of economists this century" (1987, 25).[3]

In the late 1930s, Marschak went to New York and joined the New School for Social Research. Often called the "University in Exile," the New School was a haven for European socialists (Hagemann 2017). During his time in New York, Marschak ran a seminar with fellow socialist Oskar Lange and they taught IS-LM Keynesianism to the young Italian socialist Franco Modigliani (Modigliani 2001, 19, 165).[4] Modigliani was Marschak's doctoral student, and he wrote his influential IS-LM paper under the supervision of Marschak (Modigliani 1944). As Robert Solow writes, "Franco described Marschak as his teacher and mentor; it was he who introduced Franco to the ideas of Keynes" (2005, 11).[5] Through his work with Modigliani, Marschak played a key role in spreading Keynesianism in the United States:

> "Franco Modigliani's 1944 *Econometrica* formulations, jointly with J.R. Hicks IS-LM model (Hicks 1937), set the pattern for the way Keynes's *General Theory* has been adapted and taught to a whole generation of post-war economics students. This may have had the

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D.C. In 1944, Opie helped Keynes and Harry Dexter White, a known Soviet spy, design the world monetary system at the Bretton Woods Conference. See Steil (2013).

[3] It must be stressed that Keynes invented the IS-LM model (Young and Fuller 2022, 62). It is a myth that "The IS-LM model was developed in 1937 by Nobel laureate Sir John Hicks" (Abel and Bernanke 2005, 308). In reality, Keynes invented the IS-LM model as an economic justification for non-Marxist socialism (O'Donnell 1999; Fuller 2019). The myth that Hicks invented IS-LM is pernicious, for it conceals the socialist origins of IS-LM.

[4] On Modigliani's socialism, see Modigliani (1947) and Mongiovi (2015). It is worth noting that Modigliani intentionally omitted his lengthy 1947 defense of socialism from his six volume *Collected Papers of Franco Modigliani*.

[5] Solow is incorrect when he says, "Marschak, for all his breadth of interest and sureness of taste, was not really 'into' macroeconomics" (1983, 10). Similarly, Mirowski (2012) is incorrect when he describes Cowles as an "anti-Keynesian stronghold." Dimand is correct to write, "Marschak (1951) showed commitment to expounding and developing not just macroeconomics, but specifically Keynesian macroeconomics" (2020, 38).

> merit (the great merit, many economists claim) of contributing, right at the beginning, to bringing Keynes's unorthodox stand on economic policies to be accepted in the US." (Pasinetti 2005, 36)[6]

Lange secured Marschak a position at the University of Chicago in 1943 and the two became leading figures at the Cowles Commission.[7] Marschak was the director of the Cowles Commission from 1943 to 1948. Robert W. Dimand writes, "in the 1940s Oskar Lange and Jacob Marschak took the lead at the Cowles Commission in promoting a distinctive formulation of Keynesianism" (2020, 22). Work done in connection with Marschak and Cowles led to nine Nobel awards in economics. Michel de Vroey writes,

> "Created in 1932, [the Cowles Commission] moved from Colorado to Chicago in 1939 to become the place for both high abstract theory and innovative statistical work (and trying to mix the two). The reigning attitude was one of social engineering [socialism]…. Its senior members were eminent economists, such as Jacob Marschak, Oskar Lange, and Tjalling Koopmans, but with hindsight its list of junior members was even more impressive reading, a veritable who's who of the economist profession with for instance names such as Kenneth Arrow, Gérard Debreu, Leonid Hurwicz, Franco Modigliani, and Edmond Malinvaud, in addition to [Lawrence] Klein and [Don] Patinkin." (2016, 50–51)

Politically, the Cowles Commission was a market-socialist organization. As Philip Mirowski notes, "most Cowles members in the later 40s were socialists of one stripe or another" and "the Cowles Commission was the citadel of this political movement to

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[6] Also, Marschak and Lange influenced Lawrence Klein's IS-LM approach to Keynesian economics (Klein 1966, xi). Marschak motivated Klein's influential work on Keynesian macroeconometric models: "I was stimulated by J. Marschak to build a [macro-econometric model] for the United States economy, just after completing the original version of *The Keynesian Revolution*" (1966, 227). Klein was a passionate socialist and member of the communist party (Mariano 2008, 741).

[7] See Lange (1936; 1937) for his work on the socialist calculation debate. Lange had an enormous impact on twentieth-century economics. His influential IS-LM paper (1938) was cited more frequently than Hicks's (1937) before 1950 (Rubin 2016, 301). Paul Samuelson called the IS-LM model the "Keynes Lange system" (1947, 354). Keynesians stopped citing Lange after he became a leader of the Polish communist party (Dimand 2020, 39).

forge a market socialism" (2012, 147; 2017, 74).[8] Indeed, the leading figures at Cowles were the chief defenders of socialism in the socialist calculation debate.

> "The Cowles people in contrast were a collection of European socialists and social democrats, and homegrown left-liberals…. [M]any Cowlesmen (there were indeed no women) had been active in what was known as the "socialist calculation debate." Oskar Lange, who would resign his professorship at Chicago in 1945 in order to help plan and build the communist postwar economy in Poland, had himself brought forward the Walrasian model as a planning device in his 1936 volume *On the Economic Theory of Socialism*. Thus underneath the surface interest in economic theory was an older set of arguments among economists and social philosophers about the possibility that a centralized planning system, or some kind of socialist planning model, could produce the same efficient outcomes that a competitive market economy might produce." (Düppe and Weintraub 2014, 79)[9]

This is where Harry Markowitz enters the story. Markowitz was born in Chicago in 1927 and studied at the University of Chicago (B. 1947; M.A. 1950; Ph.D. 1954). He writes, "I was a student member of the Cowles Commission at the University of Chicago" (1993, 4).[10] He attended Marschak's lectures in 1948–49 and then undertook the enormous task of editing and publishing his notes from the lectures. His notes were published in 1951 as *Income, Employment, and the Price the Level* (Marschak 1951). As Dimand says, "Marschak (1951) was the first graduate-level textbook of

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[8] W. Brus offers the following definition of market socialism: "Market socialism is a theoretical concept (model) of an economic system in which the means of production (capital) are publicly or collectively owned, and the allocation of resources follows the rules of the market (product-, labour-,capital-markets)" (1990, 164). See Huerta de Soto (1992) for an extensive critique of market socialism.

[9] For example, Arrow recalls, "I had two motivations. One was to supply a theoretical model as a basis for econometric estimation. The other was a strong interest in planning. I would have described myself as a socialist, although one that had a strong belief in the usefulness of markets. Market socialism was a widespread view" (quoted in Mirowski 2017, 76).

[10] Also, Markowitz (1999, 15n7) notes his connection to Cowles during his student days.

Keynesian macroeconomics" (2020, 28). This work contains the first textbook treatment of the Keynesian AS-AD model (Marschak 1951, 57). Since 1951, the AS-AD model has dominated macroeconomics texts. Regrettably, Marschak and his editor Markowitz are rarely mentioned in the history of macroeconomics. Thus the connection between Keynesian economics and Modern Portfolio Theory remains underappreciated.[11]

Markowitz was Marschak's doctoral student, and Marschak was the guiding force behind the project to reformulate finance. Markowitz recalls,

> "Now several years later, I am at the stage where I have to choose a dissertation. I am now at a Masters and I am working towards my Ph.D. I went to my advisor, Professor Jacob Marschak, to ask him if he had any suggestions about a dissertation topic. He was busy, so I sat out in his anteroom. There was another gentleman there and we got to talking. He was a broker and suggested that I apply mathematical statistical techniques to the stock market. So when I got in to see Professor Marschak I said, "The guy out there suggested I do a dissertation on the stock market." At the time I was a student member of the Cowles Commission and Marschak had been formerly the head of the Cowles Commission. Marschak explained that Alfred Cowles, who had endowed the Cowles Commission, was particularly interested in the application of econometric techniques to the stock market." (1993, 4)

Markowitz acknowledged in his seminal 1952 paper, "This paper is based on work done by the author while at the Cowles Commission" (1952, 77n). That year, he became a fellow at Cowles. He maintained his connection with Cowles after receiving his Ph.D. in 1954:

> "At the invitation of James Tobin I spent the 1954–55 academic year at the Cowles Foundation at Yale, on leave from RAND, writing a book

[11] Keynes would have thoroughly rejected Modern Portfolio Theory. His Platonic theory of probability puts severe limits on numerical probability, and these limits preclude Modern Portfolio Theory. But Cowles Keynesians like Klein rejected Keynes's theory of probability (Klein 1951, 446). Indeed, Keynes's logical theory of probability is almost universally rejected. See Gillies (2000, 25–49) for an overview and critique of Keynes's logical theory of probability.

that would be published in 1959 as Cowles Foundation Monograph 16, *Portfolio Selection: Efficient Diversification of Investments*. Much of the time during this period was spent writing drafts of chapters explaining the elements of mean-variance analysis." (Markowitz 1993, 5)

3. **Mean-Variance Analysis and Subjective Probability**

Markowitz inherited two fundamental elements of Modern Portfolio Theory from Marschak: (1) mean-variance analysis and (2) the subjective theory of probability. A simple example is the easiest way to illustrate mean-variance analysis. As table 1 shows, mean-variance analysis begins with a list of possible rates of return from an investment.[12] Then a probability is assigned to each possible rate of return. The list of possible rates of returns and their assigned probabilities is called the probability distribution of returns.

TABLE 1. PROBABILITY DISTRIBUTION OF RETURNS

Rate of Return	*Probability of Occurrence*
12%	18%
10	24
8	29
4	16
-4	13
Mean	7%
Variance	24.1%
Standard Deviation	4.9%

Source: Fabozzi and Markowitz (2002, 22–23).

Note: The standard deviation is the square root of the variance (Fabozzi and Markowitz 2002, 23).

[12] This paper focuses on the problem of assigning probabilities to investment returns. However, there are serious problems with using investment returns in financial decision making. Keynes introduced the rate-of-return approach to finance. Although Keynes would have thoroughly rejected it, Modern Portfolio Theory is Keynesian in the sense that it uses investment returns rather than present values. It is important to remember that Keynes's rate-of-return approach is flawed and the present-value approach is correct. See Alchian (1955) and Fuller (2020).

In mean-variance analysis, an investment is measured by the average and standard deviation of returns. These are calculated from the probability distribution of returns as indicated in table 1. Although it is typically attributed to Markowitz, Marschak originated mean-variance analysis in an October 1938 paper entitled "Money and the Theory of Assets" (Marschak 1938). As it happens the article was published in *Econometrica*, a journal funded by Alfred Cowles III.[13]

> "Marschak proposed and explored an ordinal theory of decision-making under uncertainty. He was also the first to propose that these decisions be made over the mean and the variance (or standard deviation) of the asset value. Marschak's work to formulate preferences in the now-familiar mean-variance space formed the basis for much of financial asset pricing theory…. Marschak proposed a simple approach to the consideration of the interplay between return and risk by confining its description to first moments, known as means, and second moments of returns, labeled variances and covariances. He also proposed how the variation of one asset may affect another through their covariances and their coefficient of variation…. [Marschak's] mean-variance approach emerged as the basis of Modern Portfolio Theory at the hands of his PhD supervisee and subsequent Nobel Prize winner, Harry Markowitz." (Read 2012b, 22–23)

Along with discounted cash flow analysis, mean-variance analysis is the most fundamental concept in modern finance. Dimand writes, "Markowitz's mean-variance analysis marked the beginning of the modern theory of finance" (2019, 14–15). Mean-variance analysis led to the Capital Asset Pricing Model, the Black-Scholes Model, Value-at-Risk, and many other tools of modern finance. All these tools are built on foundations laid by the socialist Marschak.

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[13] Also, *Econometrica* published the famous Keynesian IS-LM papers of Harrod (1937), Hicks (1937), and Modigliani (1944). Keynes was on the council of *Econometrica* at the time of these papers (Cowles 1937, 1943). Lange was editor when Modigliani published his paper, and Marschak was on the advisory editorial board. As noted, Lange and Marschak taught IS-LM Keynesianism to Modigliani at their seminar in New York. In his IS-LM paper, Modigliani mentions Lange some twenty times.

Probability is at the heart of mean-variance analysis. Markowitz boldly exclaimed in his seminal 1952 paper, "I believe that what is needed is essentially a 'probabilistic' reformulation of security analysis" (1952, 91).[14] As table 1 shows, the Probability of Occurrence is required to find the mean, variance, and standard deviation of returns. However, an overwhelmingly important but tragically neglected question arises: how do we determine the probability assigned to each possible rate of return? This is the most embarrassing question in Modern Portfolio Theory. Remarkably, Markowitz dodged this awkward question in his revolutionary 1952 paper: "This paper does not consider the difficult question of how investors do (or should) form their probability beliefs" (1952, 81n7).[15]

Marschak's mean-variance analysis is based on the subjective theory of probability. On July 21, 1948, Marschak presented a paper to the Cowles Commission entitled "Measurable Utility and the Theory of Assets" (Marschak 1948). The paper was published in 1950 by the Cowles outlet *Econometrica* under the name "Rational Behavior, Uncertain Prospects and Measurable Utility" (Marschak 1950). Marschak writes, "one can think of the probabilities used in the man's decisions as [subjective] 'degrees of belief' not related to specific [objective] samples in any simple way, yet obeying the usual axioms on probabilities" (1950, 113).

As with Marschak, Markowitz adopts the subjective theory of probability. He tends to stress Leonard J. Savage when endorsing the subjective theory. Still, Marschak's work came before that of Savage (Read 2012b, 23). And Markowitz acknowledges that Marschak's subjective approach influenced him (Markowitz 1993, 4; Markowitz 2013, 25). Regardless, he explicitly endorses the subjective theory of probability:

> "I was convinced by Leonard J. Savage, one of my great teachers at the University of Chicago, that a rational agent acting under

[14] Kenneth Blay writes in his introduction to Markowitz's newest work, "Modern Portfolio Theory is based on probability beliefs" (quoted in Markowitz 2013, xiv).

[15] As emphasized below, a considerable amount of excellent work had been done on these difficult questions by the time Markowitz wrote his paper. Thus the responsible course was to consider these questions before proceeding to the probability mathematics.

> uncertainty would act according to "probability beliefs" where no objective probabilities are known; and these probability beliefs or "subjective probabilities" combine exactly as do objective probabilities." (Markowitz 1991, 470)

He elaborates,

> "I was convinced in a course by Leonard J. Savage, that one should act under uncertainty as if one assigned [subjective] probability beliefs to events for which there are no objective probabilities, and should update probability beliefs according to Bayes rule. At first, I considered questions of expected utility and probability beliefs in the context of economic action in the face of risk and uncertainty. After reading F.P. Ramsey's pioneering essay, and further reflecting on Savage's arguments, I decided that the subject was the older one [epistemology] of "what do we know and how do we know it?" As explained above, I previously concluded that models of the world are never known with certainty. But we are more willing to give us [sic] some hypotheses than others. I agreed with Ramsey and Savage that [subjective] degrees of belief should be formalized in terms of the actions of a rational decision maker, i.e., a decision maker who is not omniscient, but makes no mistakes in logic or arithmetic." (Markowitz 1993, 4)[16]

As with mean-variance analysis, the subjective theory of probability is a product of socialist thought. The theory was developed independently between 1926 and 1931 by Frank P. Ramsey (1926; 1931) and Bruno de Finetti (1931; 1937). Roy Weatherford explains,

> "[T]he first subjective theory is generally held to be Frank P. Ramsey's essay 'Truth and Probability' (1931), which is self-consciously and intentionally subjectivistic in its insistence that probability

[16] See Savage's problematic book *The Foundations of Statistics* (1954). Dimand writes, "Markowitz followed Savage in positing that investors have subjective beliefs about the distribution of [investment] returns" (2009, 92). In turn, Savage followed the socialists Ramsey and de Finetti. Childers notes that Ramsey and Savage "gave more or less the same version of subjective probability" (2013, 88). In an earlier work, Markowitz (1959, 257, 307) discusses "personal probabilities" and refers to the "pioneering" work of Ramsey.

> measures the actual degree of belief of an individual…. The principal developer, defender, and disciple of subjectivistic probability theory has, therefore, been Bruno de Finetti. De Finetti's friend and ally in the English speaking world has been Leonard J. Savage. Together they made known (and somewhat respectable) the idea that there is no such thing as objective probability, only [subjective] degrees of belief." (1982, 219)[17]

Ramsey and de Finetti were socialists. As Cheryl Misak says, "[Ramsey] argued a planned economy, with state ownership and control of industry would result in more employment and greater equality and fairness than a liberal, laissez-faire one" (2020, 102).[18] Savage was associated with the market-socialists at the Cowles Commission, and he explicitly states that he based his work on de Finetti: "This book presents a theory of the foundation of statistics which is based on a personalistic [subjective] view of probability derived mainly from the work of Bruno de Finetti" (1954, 4).[19] The young Bruno de Finetti was a National Socialist in Mussolini's Italy. Indeed, he describes himself as a "faithful Blackshirt" in the last sentence of his famous 1931 paper "Probabalismo" (1931, 219). He turned to Christian socialism after the demise of Italian fascism.

> "Bruno de Finetti was a man of strong feelings, in many ways an extremist…. As a young man, he adhered to fascism [National Socialism], welcoming the nationalistic character of the movement, as well as its collectivistic tendency; he opposed the liberal idea that equilibrium can be obtained through individual profit, and heralded collective economy as a way of achieving social justice." (Galavotti 2005, 208)[20]

In summary, the foundational concepts of Modern Portfolio Theory—mean-variance analysis and the subjective theory of

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[17] The French socialist Émile Borel is sometimes considered an early subjective probability theorist (Galavotti 2005, 191). But Borel's subjective theory is "non-standard" (Weatherford 1982, 233).

[18] See Ramsey (1923) for his essay on socialism.

[19] There are also explicit references to Ramsey in Savage (1954, 7, 60, 96). Marschak (1975, 123, 128) associates the subjective theory with Ramsey, de Finetti, and Savage.

[20] Also see Lijoi and Prünster (2011, 656).

probability—have their origins in socialist thought. The socialist Marschak introduced mean-variance analysis in 1938, and he grounded it in the subjective theory of probability developed by the socialists Ramsey and de Finetti around 1931. Markowitz completed Marschak's project and published his famous paper "Portfolio Selection" in 1952. Thus Modern Portfolio Theory was born out of the socialist tradition.

4. **The Subjective Theory of Probability**

Unfortunately, those who develop, teach, and use the tools of Modern Portfolio Theory rarely think critically about the philosophy of probability. Financial professionals tend to believe philosophy is too detached from practical life to be relevant to finance. On the contrary, those interested in the future of finance and financial markets have a profound duty to study the philosophy of probability with the utmost care. The viability of the subjective theory of probability is the single most important question in modern finance. No true expert in financial science can ignore the issue of probability in finance.

According to the subjective theory, probability measures an individual's subjective degree of belief in a proposition. Donald Gillies writes, "The subjective theory identifies probability with the degree of belief of a particular individual" (2000, 1). It is important to note that subjective probabilities concern all propositions and events, including investment returns: "[The subjective theory] permits us to speak of the probability of any event whatsoever, simple or compound, unique or repetitive. If someone has an opinion on the matter, that opinion has a degree of belief, and the event therefore has a probability" (Weatherford 1982, 227).

Individual relativism is the philosophy that all opinions are equally true. It can be traced to the Greek sophist Protagoras around 450 BC: "Man is the measure of all things, of the things that are that they are, and of the things that are not that they are not" (quoted in Guthrie 1971, 183). In philosophy, individual relativism is commonly called subjectivism—the claim that truth is

relative to each individual (Lawhead 2006, 139).[21] As W.K.C. Guthrie writes, "Protagoras's was a doctrine of pure subjectivism or relativism" (1971, 184).

The subjective theory of probability is a relative theory of probability. Bruno de Finetti proclaims, "the subjective theory of probability [is] an example of the application of the relativistic mentality to such an increasingly important branch of modern mathematics as the probability calculus" (1931, 172). On the subjective theory, all probabilities are relative and equally true. Just as knowledge can never be absolute according to relativism, probability can never be absolute according to the subjective theory of probability. Gillies notes that the subjective theory "explicitly rejects enlightenment rationalism in favour of a relativistic, and even irrational, mentality" (2000, 86). Weatherford states,

> "All is subjective—nothing is absolute. In a system which defines probability as the individual's [subjective] degree of belief in a proposition, it is obvious that there can be no one answer to "what is the probability of *X*?" There are as many answers as there are beliefs, and no answer is better than any other (coherent) answer, since the individual is theoretically free to hold any opinion whatsoever.... [O]bjective probabilities are an illusion, a superstition." (1982, 233)

It has been known since at least 400 BC that relativism is self-refuting. Protagoras maintained that all opinions are equally true. But if this is true, then the opinion that it is false is true. Therefore, the proposition that all opinions are equally true is false. Aristotle observed, "For he who says that everything is true makes the statement contrary to his own also true, so that his own is not true (for the contrary statement denies that it is true)" (*Metaphysics* 1012b).[22]

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[21] Philosophical subjectivism must not be confused with economic subjectivism. Economic subjectivism holds that value is subjective. But it does not maintain that all knowledge is subjective. For example, an economic subjectivist like Mises can maintain, "It is the essential characteristic of the categories of action that they are apodictic and absolute" (1949, 196). A philosophical subjectivist must deny this.

[22] Also see Aristotle (*Metaphysics* 1009a, 1063b). On relativism as a self-refuting philosophy, see Guthrie (1978, 87) and Kenny (2010, 119). See Lawhead (2006, 149, A-7)

Relativism is widely rejected in philosophy because it is self-refuting. Anthony Gottlieb states,

> "In general, strains of relativist thinking are nowadays more often found in the writings of anthropologists, sociologists and literary critics than in the pages of professional philosophers. While a working knowledge of the diversity of human beliefs and customs seems often to encourage a sympathy for relativism, philosophers tend to be more wary of the confusions and paradoxes that lie beneath the surface of many forms of it…. Relativism, in all its forms, tends to come unstuck when applied to itself, and it has many other difficulties and obscurities too." (2000, 120)

Just like relativism, the subjective theory of probability is self-refuting. The subjective theory maintains that every person's probabilities about a proposition are equally true. If this is true, then the proposition 'the probability that the subjective theory is false is 100 percent' is true. Therefore, the subjective theory of probability is false. As with individual relativism, the subjective theory of probability must be rejected because it is self-refuting.

Marschak and Markowitz injected the virus of relativism into finance. To be sure, the subjective theory of probability must horrify anyone concerned with objectivity in finance. Sound financial analysis must be based on objective facts of reality, not subjective whims. But subjective probabilities have no objective connection to reality: "The essential feature is that [subjective] probabilities are not tied to external reality" (Weatherford 1982, 235).[23] The subjective theory is too subjective for any science, especially finance:

> "[Probabilities] are not matters of whim or opinion. The physician does not accept the patient's degree of belief that a tumor is benign; the astronomer does not ask the janitor for the probability that a cepheid variable will go nova. These probabilities are 'objective' in a way which makes 'subjective' pejorative. They are objective in a way which shows that subjective probabilities could never serve

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for an excellent overview of the argument.

[23] This point tends to infuriate advocates of the subjective theory of probability. But it is not controversial. See Bateman (1987, 100), Lawson (1988, 41), Plato (1994, 24, 245), and Galavotti (2005, 216–17, 235).

> as a basis for science. Scientists must always assume that there is a 'fact of the matter' which scientific investigation pursues and which scientific opinions approximate more or less correctly according as they are better or worse opinions. But [the subjectivist's] dictum would have it that there is no 'fact of the matter' in probability cases and therefore that all opinions are on an equal footing. This cuts at the very root of thought and removes the whole point of scientific enterprise. No, subjectivistic probability will not do for science." (Weatherford 1982, 240)[24]

5. **The Frequency Theory**

The frequency theory of probability is the correct interpretation of probability. Since Richard von Mises published his classic work *Probability, Statistics, and Truth* (1928), the frequency theory has been the official interpretation of orthodox statistics and the natural sciences (Galavotti 2017, 5, 10).[25] Moreover, the frequency approach totally dominates the practice of Modern Portfolio Theory. Although he is a subjectivist, Markowitz incoherently adopts the frequency approach when he actually applies the tools of Modern Portfolio Theory (Markowitz 1991, 472; Fabozzi and Markowitz 2002, 39). Can the frequency theory of probability bail out Modern Portfolio Theory?

According to the frequency theory, the probability of an outcome is the relative frequency of the outcome in a long series of uniform repetitions. The relative frequency is the number of times an outcome occurs divided by the total number of repetitions. For example, if a two spot comes up 166 times when a die is rolled 1,000 times, then the relative frequency is 166/1,000 = 16.6 percent.

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[24] Also see Childers (2013, 91–92).

[25] A.N. Kolmogorov provided the standard probability axiom system in *Foundations of the Theory of Probability* (1933). He does not deal with the philosophy of probability at length. However, he writes that Mises's frequency approach influenced him: "In establishing the premises necessary for the applicability of the theory of probability to the world of actual events, the author has used, in large measure, the work of R. v. Mises" (1933, 3n4). Modern interpreters agree, "relative-frequencies of occurrence of the various outcomes lie at the bottom of the Kolmogorov system" (Fine 1973, 64). Also see Plato (1994, 2, 19, 23, 205) and Gillies (2000, 125).

Richard von Mises writes, "The relative frequency of the repetition is the 'measure' of probability" (1928, vi).

Unlike the subjective theory, the frequency theory ties probability to external reality. In fact, the frequency theory holds that probability can only be determined through empirical observation. As Galavotti writes, "probability judgments are based on empirical phenomena, more precisely about repeatable phenomena of a certain kind; they do not concern personal [subjective] opinion" (2005, 72). For Modern Portfolio Theory, the frequency theory means actual market prices from the real world must be used when assigning probabilities. First, the analyst collects data on the investment's price over some period of time. Then the analyst uses these prices to calculate returns (percentage price changes). To illustrate, table 2 shows price and return data for General Electric between March 1999 and March 2003.

TABLE 2. STOCK RETURN DATA FOR GENERAL ELECTRIC (GE)

Date	*GE Price*	*GE Return*
March 2003	$25.50	6.0%
February 2003	24.05	4.7
January 2003	22.96	-5
December 2002	24.16	-9.5
.	.	.
.	.	.
.	.	.
May 1999	31.64	-3.5
April 1999	32.78	-4.8
March 1999	34.42	NA
Average (annual)		-3.4%
Standard Deviation (annual)		29.2%

Source: Brigham and Ehrhardt (2005, 153).

Note: The data for June 1999 through November 2002 are not shown but are included in all calculations.

On the frequency approach to Modern Portfolio Theory, historical returns are used to calculate the average and standard deviation of returns. Together, the average and standard deviation of

the historical returns determine the probabilities assigned to the future returns: "the inputs required for portfolio theory are generally estimated from historical [empirical] observations on the rate of returns…. The assumption is that the values obtained from historical observations are reasonable estimates for the expected returns, standard deviations, and correlations in the future" (Fabozzi and Markowitz 2002, 43–44).[26]

Significantly, the frequency theory only allows probability to be applied to outcomes of repeatable events. Gillies notes, "In the objective interpretation, probabilities are associated with repeatable conditions which have independent outcomes" (2000, 186). On the frequency theory, it is illegitimate to apply probability to singular, non-repeatable events. Jan von Plato writes, "The first requirement [of the frequency theory] limits the application of probability to repeatable events. Probability does not concern singular, unrepeatable events. The repetitions have to take place under the same conditions, which further limits the applicability of probability" (1994, 220).

Advocates of the frequency theory of probability must reject mean-variance analysis and, with it, Modern Portfolio Theory. The frequency theory prohibits the application of probability to investment returns because they are not outcomes of repeatable events. Every financial professional knows that each day in the market is different; market conditions are not uniform over time. A data set of historical returns is an empirical series. But each return in the data set is an outcome of non-repeatable events. To paraphrase Gillies (2000, 192), it is impossible to introduce a satisfactory notion of an independent repetition of the state of financial markets, and we cannot therefore use objective probabilities in finance. Since investment returns always concern singular events, Modern Portfolio Theory is incompatible with the frequency theory of probability.

Did Marschak and Markowitz realize there are grave objections to applying numerical probability to investment returns? They certainly did. Frank Knight distinguished risk and

[26] This assumption proved disastrous in the Financial Crisis of 2008. As Blay admits, "correlations between most asset pairs coalesced toward one" (quoted in Markowitz 2013, xv). Also see note 31 below.

uncertainty in his classic work *Risk, Uncertainty and Profit* (1921).[27] He introduced the distinction between risk and uncertainty to show that uncertainty, not risk, is the source of profit and loss—and by extension, investment returns.

> "The essential fact is that "risk" means in some cases a quantity susceptible of measurement, while at other times it is something distinctly not of this character; and there are far-reaching and crucial differences in the bearings of the phenomenon depending on which of the two is really present and operating. . . . It will appear that a measurable uncertainty, or "risk" proper, as we shall use the term, is so far different from an unmeasurable one that it is not in effect an uncertainty at all. We shall accordingly restrict the term "uncertainty" to cases of the nonquantitative type. It is this "true" uncertainty, and not risk, as has been argued, which forms the basis of a valid theory of profit [investment returns]." (Knight 1921, 19–20)[28]

In terms of the frequency theory, risk concerns repeatable events, and uncertainty concerns non-repeatable events. Knight says, "The practical difference between the two categories, risk and uncertainty, is that in the former the distribution of the outcome in a group of instances is known … while in the case of uncertainty this is not true, the reason being in general that it is impossible to form a group of instances, because the situation dealt with is in a high degree unique" (1921, 233). As Knight stressed, "[investment returns] deal with situations which are far too unique, generally speaking, for any sort of statistical tabulation to have any value for guidance. The conception of an objectively measurable probability or chance is simply inapplicable" (1921, 231).

Mean-variance analysis brazenly ignores Knight's crucial distinction between risk and uncertainty. Importantly, Marschak (1937, 81) was aware of the distinction before he introduced

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[27] Ludwig von Mises (1949, 106–15) makes a similar distinction, but he uses the term class probability for risk and case probability for uncertainty. On class and case probability, see Rothbard (1956, 309n40; 1962, 552–55) and Hoppe (2007). See Huerta de Soto (1992, 38n12) for a useful table on class and case probability.

[28] Markowitz (2020, 193) is aware of this important passage.

mean-variance analysis. He wrote in August 1938, "We appreciate the importance of Professor Knight's distinction between risk and uncertainty" (Makower and Marschak 1938, 271n1). In his landmark October 1938 paper on mean-variance analysis, Marschak noted "Professor Knight's important distinction between 'risk' and 'uncertainty'" (1938, 324).[29] He admitted in 1960, "Frank Knight showed that relevant future events are, in the main, not repetitive" (1960, 541). Although he conveniently neglected Knight for seven decades, Markowitz finally acknowledged that *Risk, Uncertainty and Profit* is a "classic book" (2020, 193, 218).[30] Hence the founders of Modern Portfolio Theory knew that applying numerical probability in finance was considered "fatally misleading" (Knight 1921, 231).

Frequentists must view Modern Portfolio Theory as the most irresponsible and dangerous abuse of probability in the history of humankind.[31] Rather than accepting the hard truth that probability is not suitable for finance, Marschak and Markowitz revolted against the limits of probability. Their followers missed a vital point: the frequentists developed the strict limits of the frequency theory to combat charlatanism (Gillies 2000, 97). The frequentists were appalled that charlatans were abusing probability to deceive others. Thus they framed limits on probability in a noble effort to prevent charlatans from preying upon the ignorance of unsuspecting victims. In short, the frequentists designed limits on

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[29] Marschak deliberately worked to undercut Knight's influence at the University of Chicago. For example, in 1951, "Marschak opposed creating a Frank Knight chaired professorship at Chicago" (Mirowski 2017, 75).

[30] Markowitz (1952; 1959) never mentions Knight. Also, there is no mention of Knight in Markowitz's *Selected Writings* (2008). This is curious, for Markowitz must have studied *Risk, Uncertainty and Profit* as a student at the University of Chicago. This also applies to Savage (1954).

[31] The Crisis of 2008 should have taught the finance profession to abandon Modern Portfolio Theory. Blay's attitude is typical: "The financial collapse of 2008 and 2009 resulted in a tremendous and sudden decline in worldwide wealth as correlations between most asset pairs coalesced toward one…. This trauma precipitated a serious questioning of the efficacy of MPT diversified portfolios…. Like many practitioners, my colleagues and I began wondering whether the MPT paradigm had run its course…. [W]e are today, more than ever, vociferous and enthusiastic advocates of MPT" (quoted in Markowitz 2013, xv).

probability for the express purpose of preventing hazardous constructions like Modern Portfolio Theory.

6. **Conclusion**

Modern Portfolio Theory was doomed from the start. The frequency theory of probability is correct. However, it prohibits the application of probability to investment returns. For this reason, Marschak and Markowitz's attempt at a "probabilistic reformulation" of finance was destined to be a scientific failure. So what drove them to error? Ludwig von Mises writes,

> "[R]ecourse to the notion of rationalization provides a psychological description of the incentives which impelled a man or a group of men to formulate a theorem or a whole theory. But it does not predicate anything about the validity or invalidity of the theory advanced. If it is proved that the theory concerned is untenable, the notion of rationalization is a psychological interpretation of the causes which made its authors liable to error." (1949, 79)

Socialism made Marschak and Markowitz liable to error. Mises used the theory of classical finance to show that socialism makes economic calculation impossible. In the terminology of modern finance, Mises showed it is impossible to make net present value calculations in a socialist society. This devastated socialism, for "The method of 'net present value' is the most important tool in modern finance" (Goetzmann 2016, 243).[32] Socialists like Marschak could not refute Mises within the framework of classical finance. So they revolted against it. In the service of socialism, Marschak and Markowitz attempted to undercut classical finance with a "probabilistic reformulation" of the science. William N. Goetzmann observes,

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[32] Ivo Welch is correct to write, "NPV is the most important building block in finance" (2009, 67). See Brigham and Ehrhardt (2005, 349–50) for a standard introduction to the net present value.

> "What is remarkable about Markowitz's method is that he completely disregarded the tenets of fundamental investing and deep research. He assumed that this information was already impounded in the prices and expected returns of the stocks. The only information a Markowitz-type investor needed was statistical: the expected return for the stock, the volatility [standard deviation] of the stock (Regnault's "vibration"), and the correlation, or co-movement of each stock with every other one. He turned investment management from a profession based on deep research on companies into a mathematical exercise…. The abstract scientist, armed with computer programs and linear algebra, eclipsed the heroic, loner, fundamental analyst…. [T]he Markowitz model has become the primary tool used by virtually all institutional portfolio managers in the world." (2016, 506–7)

Modern Portfolio Theory is a product of the socialist revolt against reason. Mises writes in his classic chapter "The Revolt Against Reason,"

> "The economists had entirely demolished the fantastic delusions of the socialist utopians…. The socialists were absolutely unable to raise any objection to the devastating criticism of their schemes and to advance any argument in their favor. It seemed as if socialism was dead forever. Only one way could lead the socialists out of this impasse. They could attack logic and reason." (1949, 73–74)[33]

Intransigent socialists must reject logic and reason. In their revolt against reason, the socialists Marx and Engels advocated the philosophy of polylogism (Mises 1949, 75–89). By contrast, the socialists Ramsey and de Finetti turned to the individual relativism of Protagoras in their revolt against reason. As W.T. Jones explains, Protagoras's relativism is the opposite of reason and science:

> "If neither reason nor perception yields the truth about the world, Protagoras argued, objective knowledge of a public reality is quite impossible. "Man," he said, "is the measure of all things, of things

[33] Also see Mises (1922, 60).

> that are that they are and of things that are not that they are not." This was, of course, a complete rejection of the whole philosophic and scientific enterprise as it had been conceived since Thales. For everyone since Thales' day had held that (1) there is a public, objective reality and that (2) this reality is intelligible, that is, it can be understood by the human mind. All this Protagoras in effect denied." (1952, 60)

In the words of Anthony Kenny,

> "Protagoras is sometimes described as a sceptic. In one way this is an odd description. A sceptic is someone who thinks the discovery of truth is difficult, perhaps impossible. For Protagoras it is all too easy: you only have to frame a belief and, hey presto, it is true. But from the point of view of someone like Democritus [and Aristotle], the replacement of a universal, objective concept of truth with a relative one is itself a very deep form of scepticism. The only kind of truth really worth seeking is, for a relativist, impossible to discover because it does not exist." (2010, 120)

Socialism and relativism go hand in hand. Bruno de Finetti proclaims in his famous 1931 paper on probability, "fascism [National Socialism] represents the relativistic attitude in politics" (1931, 223n48). Indeed, in the same paper, the socialist de Finetti endorses this statement from Benito Mussolini:

> "Fascism is a super-relativistic movement … Everything I have said and done in these last years is relativism by intuition. If relativism signifies the end of faith in science, the decay of that myth, 'science,' conceived as the discovery of absolute truth, I can boast of having applied relativism to the analysis of socialism. If relativism signifies contempt for fixed categories and men who claim to be the bearers of an external objective truth . . . then there is nothing more relativistic than Fascist attitudes and activity." (quoted in Neumann 1942, 462–63)[34]

[34] Mussolini was a Marxist before he became a national socialist: "Marx had a very big influence on [Mussolini]…. In later years, fascists and communists were anxious to deny that he had ever been a marxist, but from 1904 to 1914 he was accepted by

Just as they must rebel against all pro-reason philosophies, obstinate socialists must revolt against the frequency theory of probability. The frequency theory emerged from the pro-reason tradition dating back to Aristotle (Nagel 1939, 18; Weatherford 1982, 13, 144). Being socialists, Ramsey and de Finetti had to reject the frequency theory and thus developed the subjective theory. The subjective theory is the application of the anti-reason, anti-science philosophy of individual relativism to probability. Just like the sophist Protagoras, however, subjective probability theorists refute themselves. If every belief is true, then the belief 'the subjective theory of probability is incorrect' is true.

In their revolt against reason, Marschak and Markowitz attempted to reformulate finance with the subjective theory of probability. By using the subjective theory, however, they rooted Modern Portfolio Theory in individual relativism (Markowitz 2006).[35] Modern Portfolio Theory is the application of individual relativism to financial decision-making. Thus it is an anti-reason, anti-science approach to finance. Unfortunately, the profession did not look past the shiny mathematical exterior of the theory to its rotten relativistic core. The profession did not realize that Marschak and Markowitz were ensnared in the trap of relativism. Specifically, advocates of Modern Portfolio Theory must maintain that every belief is true. But if every belief is true, then the belief 'Modern Portfolio Theory is incorrect' is true.

The finance profession must abandon Modern Portfolio Theory. Everyone agrees it was a revolution in finance. More precisely, it was a socialist revolution against the time-honored science of classical finance. As Ludwig von Mises always stressed, "Our civilization is inseparably linked with our methods of economic calculation. It would perish if we were to abandon this most precious intellectual tool of acting" (1949, 231). Socialism makes

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others as one … According to Mussolini, 'Marx was the greatest of all theorists of socialism'" (Smith 1982, 7).

[35] Markowitz admits an interest in epistemology: "When I was in high school I read a lot in philosophy…. I was especially interested in 'what do we know,' and 'how do we know it,' and the 'uncertainty of it all'" (2002, 3). This might suggest he should have been aware of the errors of relativism.

economic calculation impossible and thereby threatens civilization. Analogously, Modern Portfolio Theory threatens civilization by corrupting economic calculation. The finance profession must renounce the anti-science of Modern Portfolio Theory and return to the true science of classical finance.

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